
ABSTRACT

New mathematical models of electromagnetic field of microwave rectangular and circular waveguides have been established in polar coordinate system. Ritz and Galerkin methods for solving mathematical models have been applied and the algorithms of these methods have been applied to address the issue under consideration. As a result of solving mathematical models curves of ekvipotential lines of electromagnetic field of microwave rectangular and round waveguide have been established. Built-in curves created the opportunity to monitor the distribution of electromagnetic field of microwave rectangular and circular waveguide. The developed methods are differentiated by their universality. Thus, this technique can be applied to the other extreme microwave devices.

KEYWORDS: microwave, rectangular waveguide, circular waveguide, electromagnetic fields, Ritz method, Galerkin method.

INTRODUCTION

At present, microwave devices are widely used in the production [1-3]. Therefore, the improve of constructive, technical and operating parameters and the design of new devices which have more optimal constructive sizes are very important and actual both scientific point of view and in terms of production [4-7]. For this reason, the introduced article has been dedicated to the mathematical modeling of electromagnetic field of microwave range devices. In the article, the structure of microwave rectangular and round waveguides is more complicated than of the electromagnetic field mathematical modeling of the E-type and H-type is considered. Developed new algorithms have been based on a mathematical point of view, accurate and complete [8-10].

Two main aims are touched when this article is prepared: 1. For mathematical modeling of microwave devices, available analytical, numerical methods are analyzed and compared and a more effective algorithm is worked out. 2. Developed algorithms are applied and microwave devices are worked out which are more perfect.

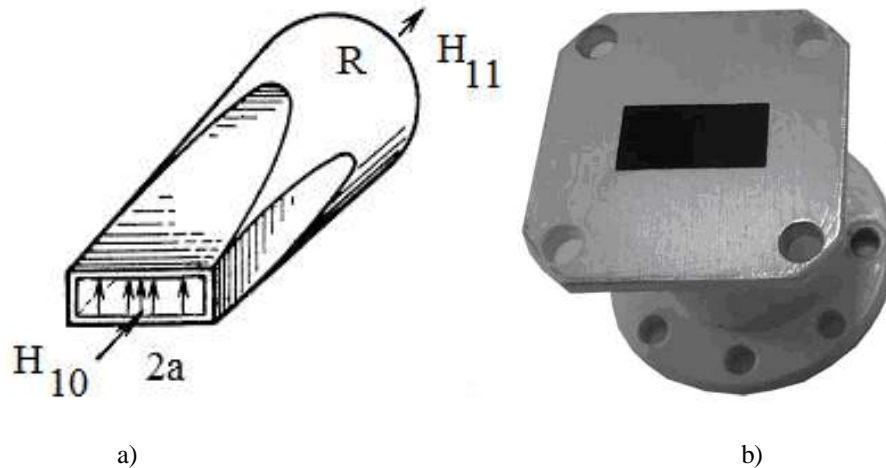


Figure 1. Transition from microwave rectangular waveguide to circular (a) and spatial view (b).

MATHEMATICAL MODELS OF ELECTROMAGNETIC FIELD OF MICROWAVE CIRCULAR WAVEGUIDE

If we point out prevalence of electromagnetic field in microwave waveguide with oblast S , we can indicate the integral equation that express the field as follows [11, 12]:

$$J = \int_s \left[\left(\frac{\partial E}{\partial x} \right)^2 + \left(\frac{\partial E}{\partial y} \right)^2 + 2F(x, y)E \right] ds, \quad (1)$$

here $F(x, y)$ is a given function.

We can call that as functional, because the price of (1) integral depends on the function E . We should accept for clarifying the physical nature of the problem, E function is a minimum of (1) functional. Therefore, let's look at the following function [13-15]:

$$E(x, y) + \alpha \eta(x, y),$$

here α – is a very small proportion, $\eta(x, y)$ is a continuous function and in oblast S the first two differential of itself is approaching to zero. Therefore, under the condition of $\alpha \neq 0$

$$J(E + \alpha \eta) \geq J(E).$$

So, when (1) functional becomes $\alpha = 0$, it becomes as a minimum price, so,

$$\left. \frac{dJ(E + \alpha \eta)}{d\alpha} \right|_{\alpha=0} = 0.$$

Let's count this differential:

$$\frac{dJ(E + \alpha \eta)}{d\alpha} = \frac{d}{d\alpha} \int_s \left[\left(\frac{\partial E}{\partial x} + \alpha \frac{\partial \eta}{\partial x} \right)^2 + \left(\frac{\partial E}{\partial y} + \alpha \frac{\partial \eta}{\partial y} \right)^2 + 2F(x, y)(E + \alpha \eta) \right] ds.$$

Now if admit that there is $\alpha = 0$, we shall determine:

$$\int_s \left[\frac{\partial E}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial E}{\partial y} \frac{\partial \eta}{\partial y} + F\eta \right] ds = 0. \quad (2)$$

If we turn (2) integral, we can write as follows:

$$\int_s \left[\frac{\partial}{\partial x} \left(\eta \frac{\partial E}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta \frac{\partial E}{\partial y} \right) \right] ds - \int_s \eta \left[\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} - F \right] ds = 0,$$

or

$$\int_S \operatorname{div}(\eta \nabla E) ds - \int_S \eta (\Delta E - F) ds = 0. \quad (3)$$

Due to Ostrogradski theorem, we can write:

$$\int_S \operatorname{div}(\eta \nabla E) ds = \int_S \eta \frac{\partial E}{\partial n} dl,$$

here C - the limited contour with the surface S , \vec{n} - the normal directed to contour C .

It should be noted that because η parameter is approaching to zero along C contour, we receive

$$\int_S \operatorname{div}(\eta \nabla E) ds = 0,$$

$$\int_S \eta (\nabla E - F) ds = 0.$$

Acquired expression does not depend on η function and under the following condition is true:

$$\Delta E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = F(x, y). \quad (4)$$

So it can be concluded that, (1) the minimum of integral (4) is equivalent to the solution of the Poisson equation. It should be noted that (1) integral is minimum and it is necessary that this is a equivalent to the solution of the equation (4).

In particular, when $F(x, y) = 0$ (1) integral takes the form of integral of Dirichlet [5-9]. Therefore, the solving of the Laplace equation is equivalent with determination of minimum function of integral of Dirichlet. In general the determination of the minimum function of (1) integral is fulfilled by applying the methods of Ritz and Galerkin. Therefore, let's apply these methods for modeling of electromagnetic field of microwave rectangular and circular waveguides.

APPLICATION OF THE METHOD OF RITZ AND THE GALERKIN

It should be accepted that $E(x, y)$ is an exact solution of the problem and minimum of (1) functional is

$J(U) = m$. If we are able to find out function of $\vec{E}(x, y)$ which meet the conditions of the border for E , then built \vec{E} function will be approached to a genuine solution of E . On the other hand, if we have determined the sequence of the \vec{E}_n function, ie, $J(\vec{E}_n)_{n \rightarrow \infty} \rightarrow m$, then this sequence will assemble up to the settlement of the E .

Thus, according to the method of Ritz \vec{E}_n function will depend on the following n parameters in the treated issue:

$$\vec{E}_n = F(x, y, a_1, a_2, \dots, a_n).$$

(1) if we put \vec{E}_n function instead of E function and fulfill integration operation, then the result will be a function of the a_k ($k = 1, 2, \dots, n$) parameters. So that,

$$J(\vec{E}_n) = J(a_1, a_2, \dots, a_n).$$

The issue is correlated with appointing of a minimum of J functional, then a_k parameters should be as the following equations:

$$\frac{\partial J}{\partial a_k} = 0 \quad (k = 1, 2, \dots, n).$$

It should be noted that the \bar{E}_n functions are full under the following feature:

$$\lim_{n \rightarrow \infty} J(\bar{E}_n) = J(E) = m.$$

According to the method of Ritz, issue of minimization should be sought in the form of linear functions which depend on a_k parameters:

$$\bar{E}_n = \sum_{k=1}^n \alpha_k \gamma_k(x, y). \quad (6)$$

It should be noted that E parameter fulfills the homogeneous boundary conditions along C contour. Otherwise, you should find any of the $E_1(x, y)$ function, this function must carry out the following condition:

$$E_1(x, y) = E(x, y).$$

Therefore, it is necessary to include the new $V(x, y)$ function. This function must fulfill the following condition:

$$E_1(x, y) = V(x, y) + E_1(x, y).$$

So that,

$$\Delta V = F - \Delta E_1.$$

If $\omega(x, y) = 0$ is along the contour C , then you can write the following as a ω_k functional system:

$$\gamma_1 = \omega, \gamma_2 = \omega x, \gamma_3 = \omega y, \gamma_4 = \omega x^2, \gamma_5 = \omega xy, \gamma_6 = \omega y^2.$$

If $F(x, y) = 0$ is along the contour C , then we can choose $\omega(x, y)$ function as follows [10-15]:

$$\omega(x, y) = \pm F(x, y).$$

At the same time the solving of (4) Poisson equation will be (6) in the form as follows:

$$\begin{aligned} J(\bar{E}_n) &= \int_S \left[\left(\frac{\partial \bar{E}_n}{\partial x} \right)^2 + \left(\frac{\partial \bar{E}_n}{\partial y} \right)^2 + 2F \bar{E}_n \right] ds = \int_S \left[\left(\sum_{k=1}^n a_k \frac{\partial \varphi_k}{\partial x} \right)^2 + \left(\sum_{k=1}^n a_k \frac{\partial \varphi_k}{\partial y} \right)^2 + 2F \sum_{k=1}^n a_k \varphi_k \right] ds = \\ &= \sum_{s=1}^n \sum_{k=1}^n \alpha_{ks} a_k a_s + 2 \sum_{k=1}^n \beta_k a_k, \end{aligned} \quad (7)$$

$$\text{here } \alpha_{ks} = \alpha_{sk} = \int_S \left(\frac{\partial \varphi_k}{\partial x} \frac{\partial \varphi_s}{\partial x} + \frac{\partial \varphi_k}{\partial y} \frac{\partial \varphi_s}{\partial y} \right) ds,$$

$$\beta_k = \int_S F \varphi_k ds. \quad (8)$$

The system of equations can be written as follows which define a_k parameters:

$$\frac{1}{2} \frac{\partial J(\bar{E}_n)}{\partial a_k} = \sum_{s=1}^n \alpha_{ks} a_s + \beta_k = 0 \quad (k = 1, 2, \dots, n), \quad (9)$$

or

$$\frac{1}{2} \frac{\partial J(\bar{E}_n)}{\partial a_k} = \int_S \left(\frac{\partial \bar{E}_n}{\partial x} \frac{\partial \varphi_k}{\partial x} + \frac{\partial \bar{E}_n}{\partial y} \frac{\partial \varphi_k}{\partial y} + F \varphi_k \right) ds = 0 \quad (k = 1, 2, \dots, n). \quad (10)$$

We should point out solving of (9) or (10) system with a_k^n . Then we will receive basis on the method of Ritz :

$$\bar{E}_n(x, y) = \sum_{k=1}^n a_k^n \varphi_k(x, y). \quad (11)$$

According to the method of Galerkin, solving of the problem is sought in the form of linear functions which depend on a_k . But in this case the equations will be in another form of for their determination. In this case, if we do conversions in the (10) equation, we will receive:

$$\int_s \left(\frac{\partial \bar{E}_n}{\partial x} \frac{\partial \varphi_k}{\partial x} + \frac{\partial \bar{E}_n}{\partial y} \frac{\partial \varphi_k}{\partial y} + F \varphi_k \right) ds = \int_s \left[\frac{\partial}{\partial x} \left(\varphi_k \frac{\partial \bar{E}_n}{\partial x} + \frac{\partial}{\partial y} \left(\varphi_k \frac{\partial \bar{E}_n}{\partial y} \right) \right) \right] ds - \int_s \left(\frac{\partial^2 \bar{E}_n}{\partial x^2} + \frac{\partial^2 \bar{E}_n}{\partial y^2} - F \right) \varphi_k ds =$$

$$= \int_c \varphi_k \frac{\partial \bar{E}_n}{\partial \nu} dl - \int_s \left(\frac{\partial^2 \bar{E}_n}{\partial x^2} + \frac{\partial^2 \bar{E}_n}{\partial y^2} - F \right) \varphi_k ds = 0,$$

here $\nu - C$ is outer normal towards the contour.

If the sought solution pays homogeneous boundary conditions, then γ_k function will be equal to zero along the contour C . In this case, we will receive:

$$\int_s \left(\frac{\partial^2 \bar{E}_n}{\partial x^2} + \frac{\partial^2 \bar{E}_n}{\partial y^2} - F \right) \varphi_k ds = 0 \quad (k = 1, 2, \dots, n). \quad (12)$$

If the equation is in the form of $L(E) = 0$, then according to the method of Galerkin approaching to n a_k ratios are as follows:

$$\bar{E}_n(x, y) = \sum_{k=1}^n a_k^n \varphi_k(x, y),$$

$$\int_s L(\bar{E}_n) \varphi_k ds = 0 \quad (k = 1, 2, \dots, n). \quad (13)$$

Now, we should solve mathematical models for a specific device. One side of the device is circular waveguide as shown in figure 1 and the other side is the rectangular waveguide which one of them work in H_{11} type, other works on H_{10} type waves. Radius of circular waveguide is R , side of rectangular waveguide is $2a$ (figure 1). The intensity of electric field is E_0 in circular waveguide. The issue should be resolved with integration of equation $\Delta E = 0$. First of all, we should make boundary conditions as a homogeneous form. Because of this, we should receive as follows:

$$E = V + E_1.$$

At the same time, we choose E_1 function so that approaches to E_0 in circular part of the waveguide, to zero in rectangular section. Let's look at the issue in (ρ, γ) polar system. In this case, we can write E_1 function as follows:

$$E_1 = E_0 \frac{\rho - F(\gamma)}{R - F(\gamma)},$$

here

$$F(\varphi) = \begin{cases} \frac{a}{\cos \varphi}, & -\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{4}, \\ -\frac{a}{\cos \varphi}, & \frac{3\pi}{4} \leq \varphi \leq \frac{5\pi}{4}, \\ \frac{a}{\sin \varphi}, & \frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}, \\ -\frac{a}{\sin \varphi}, & \frac{5\pi}{4} \leq \varphi \leq \frac{7\pi}{4}. \end{cases}$$

In this case, let's solve the following equation for the function V under homogeneous boundary conditions:

$$\Delta V = -\Delta E_1.$$

If we consider, we will receive:

$$V_n = \sum_{k=1}^n a_k \varphi_k(\rho, \varphi).$$

Then the solving of the equation will be in approximation n as follows:

$$E = E_1 + V_n.$$

According to (12) testimony we can write for the determination of the coefficients a_k :

$$\int_S \sum_{k=1}^n a_k \varphi_m \Delta \varphi_k ds + \int_S \varphi_m \Delta E_1 ds = 0 \quad (m = 1, 2, \dots, n),$$

or

$$\sum_{k=1}^n a_k \int_0^{\pi/4} d\varphi \int_R^{\frac{a}{\cos \varphi}} \varphi_m \Delta \varphi_k \rho d\rho + \int_0^{\pi/4} d\varphi \int_R^{\frac{a}{\cos \varphi}} \varphi_m \Delta E_1 \rho d\rho = 0.$$

We can write the following functions under a symmetry condition of field as a function φ_k which approaches to zero in computing region:

$$\begin{cases} \varphi_1(\rho, \varphi) = (\rho - R) [\rho - F(\varphi)], \\ \varphi_2(\rho, \varphi) = \varphi_1(\rho, \varphi) \rho^2 \cos^2 \varphi, \\ \varphi_3(\rho, \varphi) = \varphi_1(\rho, \varphi) \rho^2 \sin^2 \varphi, \\ \varphi_4(\rho, \varphi) = \varphi_1(\rho, \varphi) \rho^4 \cos^2 \varphi \sin^2 \varphi. \\ \dots \end{cases}$$

In concrete case let's look at solving of the function φ_1 for 1st approximation. At this time we shall get for calculation of the ratio of a_1 :

$$a_1 \int_0^{\pi/4} d\varphi \int_R^{\frac{a}{\cos \varphi}} \varphi_1 \Delta \varphi_1 \rho d\rho + \int_0^{\pi/4} d\varphi \int_R^{\frac{a}{\cos \varphi}} \varphi_1 \Delta E_1 \rho d\rho = 0.$$

We should consider for Laplace to be as follows in polar coordinate system:

$$\Delta\psi = \frac{\partial}{\partial\rho} \left(\rho \frac{\partial\psi}{\partial\rho} \right) + \frac{1}{\rho} \frac{\partial^2\psi}{\partial\varphi^2}.$$

Then we will get a ratio of a_1 :

$$a_1 = -\frac{\alpha}{a^2},$$

$$\text{here } \alpha = \frac{p(1,11-1,42p+0,524p^2)}{0,518-3,73p-6,36p^2-3,11p^3-0,514p^4+0,734p^5}, p = \frac{R}{a}.$$

RESULTS AND DISCUSSION

We will receive for intensity in the first approximation intensity:

$$\frac{E_1}{E_0} = \frac{\frac{\rho}{a} - \frac{1}{\cos\varphi}}{p - \frac{1}{\cos\varphi}} + \alpha \left(\frac{\rho}{a} - \frac{1}{\cos\varphi} \right) \left(\frac{\rho}{a} - p \right), \quad \left(-\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{4} \right).$$

Jerky curve complies with part of the ekvipotential line in square $0 \leq \varphi \leq \pi/2$ in figure 2. When $p = \frac{R}{a} = 0,5$

becomes the calculation is carried out.

Clearly, the first approximation is non-satisfactory for taking advantage. Therefore, the following is accepted for the next approximation.

$$E = E_1 + a_1\varphi_1(\rho, \varphi) + a_2\varphi_2(\rho, \varphi),$$

here φ_1 is the same, $\varphi_2 = \varphi_1 \cos 2\varphi$.

This type of solution has been chosen for simplifying the calculations and φ_2 and φ_3 functions have been connected in (14) equation. At the same time, ρ^2 pairs of factors have been taken. Thus, if we detained factor of ρ^2 , then, we would have calculated the interval 12. This would also make difficult solving of the issue. We will receive the system of following equations for determining ratios of a_1 and a_2 :

$$\begin{cases} a_1 \int_s \varphi_1 \Delta\varphi_1 ds + a_2 \int_s \varphi_1 \Delta\varphi_2 ds = - \int_s \varphi_1 \Delta E_1 ds, \\ a_1 \int_s \varphi_2 \Delta\varphi_1 ds + a_2 \int_s \varphi_2 \Delta\varphi_2 ds = - \int_s \varphi_2 \Delta E_1 ds. \end{cases}$$

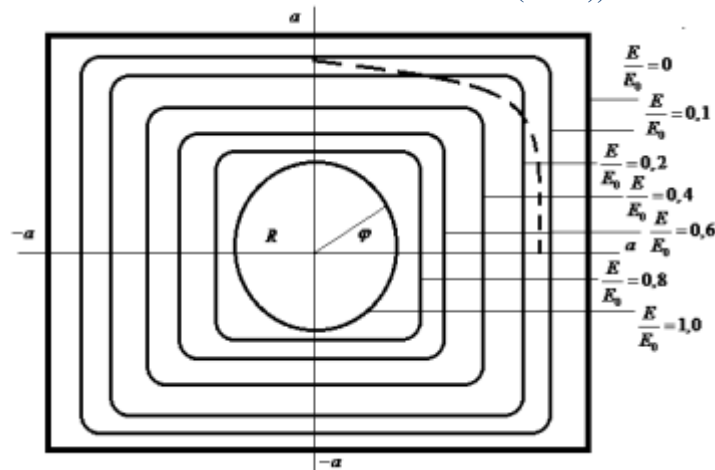


Figure 2. Distribution of electromagnetic field from microwave rectangular waveguide to circular in transition devices

By analogy, if the calculation is shown above, we will receive:

$$a_1\psi_{11}(p) + a_2\psi_{12}(p) = \psi_{13}(p),$$

$$a_1\psi_{21}(p) + a_2\psi_{22}(p) = \psi_{23}(p),$$

here $p = \frac{R}{a}$ and

$$\psi_{11}(p) = a^2(0,52 - 3,73p + 6,36p^2 - 3,11p^3 - 0,51p^4 + 0,73p^5),$$

$$\psi_{12}(p) = -a^2(2,93 - 9,52p + 12,08p^2 - 9,39p^3 + 5,08p^4 - p^5),$$

$$\psi_{13}(p) = -p(1,11 - 1,43p + 0,52p^2),$$

$$\psi_{21}(p) = a^2(0,08 - 1,17p + 1,43p^2 - 1,27p^3 - 1,32p^4 + 0,47p^5),$$

$$\psi_{22}(p) = -a^2(1,09 - 3,94p + 5,07p^2 - 5,26p^3 + 1,89p^4 - 0,78p^5),$$

$$\psi_{23}(p) = -p(0,51 - 0,51p + 0,43p^2).$$

If we assume $p = 0,5$, we will receive:

$$\psi_{11} = -0,15a^2, \psi_{12} = -0,31a^2, \psi_{13} = -0,26,$$

$$\psi_{21} = -0,38a^2, \psi_{22} = 0,17a^2, \psi_{23} = -0,18,$$

$$a_1 = \frac{0,708}{a^2}, a_2 = \frac{0,508}{a^2}.$$

Thus, the solution sought is as follows:

$$\frac{E}{E_0} = \frac{\rho - F}{R - F} + (0,708 + 0,508 \cos 2\phi) \frac{(\rho - F)(\rho - R)}{a^2}.$$

Equipotential lines of the field determining by this formula have been shown with curves by complete lines in figure 2.

CONCLUSION



1. New mathematical models of electromagnetic field of microwave rectangular and circular waveguides have been established in polar coordinate system.

2. Ritz and Galerkin methods for solving mathematical models have been applied and the algorithms of these methods have been developed for solving the issue.
3. As a result of the settlement of the developed mathematical models, curves of ekvipotensial lines of electromagnetic field of microwave rectangular and circular waveguides have been established.
4. Built-in curves have been created the opportunity to monitor to the distribution of electromagnetic field of microwave rectangular and circular waveguides.
5. The developed methods are differentiated by their universality. Thus, this technique can be applied to the other extreme microwave devices.

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